Two-stage spline approximation to scattered data

The NAG Library's capabilities for surface fitting have been extended at Mark 24.

Our Curve and Surface Fitting Chapter now includes a routine for computing a spline approximation to a set of scattered data. Typical real-world input (from, say, geosciences, data mining, or medical imaging) can contain millions of points: this routine is designed to easily accommodate large data sets. For sufficiently-dense input data consisting of n points, n-linear complexity and memory usage can be attained.

The new fitting routine uses least-squares polynomials to compute local spline approximations over a uniform triangulation of the approximation domain. These approximations are then C^1 smoothed to form the final spline. Two evaluators, for a vector and a mesh, have been provided for computing values of the spline.

The Franke function

$$\begin{aligned} f(x,y) \quad = \quad & 0.75\exp\left(-\left((9x-2)^2+(9y-2)^2\right)/4\right)+ \\ & 0.75\exp\left(-(9x+1)^2/49-(9y+1)/10\right)+ \\ & 0.5\exp\left(-\left((9x-7)^2+(9y-3)^2\right)/4\right)- \\ & 0.2\exp\left(-(9x-4)^2-(9y-7)^2\right) \end{aligned}$$

is widely used for demonstrating surface-fitting methods. Figure 1 shows the surface generated by the new routine when given a random set of input (x, y) points on the unit square.
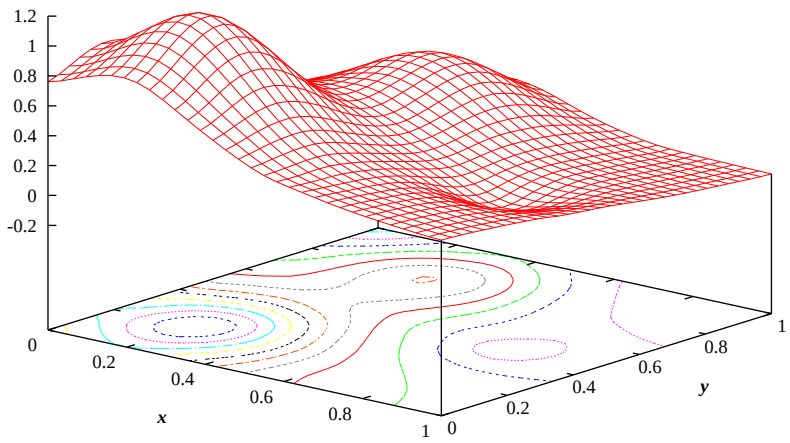
Figure 1: Calculation and Evaluation of Bivariate Spline Fit from Scattered Data using Two-Stage Approximation