Confluent Hypergeometric Function ${}_1F_1(a;b;x)$ (S22BA, S22BB)

The routines S22BA and S22BB, new at Mark 24, provide the functionality to calculate the confluent hypergeometric function ${}_1F_1(a;b;x)$, also known as Kummer's function $M(a,b,x)$. This has a wide variety of applications, including CIR processes and pricing Asian options. Many special functions are also expressible as special cases of ${}_1F_1$, including the incomplete gamma function, Bessel functions and Laguerre polynomials.

$M(a,b,x)$ is one of the independent solutions to the differential equation,

$$x\frac{d^2M(a,b,x)}{dx^2} + (b-x)\frac{dM(a,b,x)}{dx} - aM(a,b,x), \tag{1}$$

and can be defined via the power series,

$$M(a,b,x) = \sum_{j=0}^{\infty} \frac{(a)_j x^j}{(b)_j j!}, \tag{2}$$

where $(\alpha)_j = 1(\alpha)(\alpha+1)\dots(\alpha+j-1)$ is the rising Pockhammer function of $\alpha \in \{a,b\}$.

S22BA returns the value M directly given the values a, b and x. S22BB returns the solution in the form $M(a,b,x) = m_f \times 2^{m_s}$. $M(a,b,x)$ rapidly exceeds standard precision limits for even moderate values of the parameters a, b and x ($\sim O(100)$), and as such the availability of the fractional component m_f and scale m_s allows for meaningful results to be returned over much greater ranges. Figure 1 shows $M(-150 \le a \le 150, -150 < b < 150, x = 25)$, plotted as $\frac{M}{|M|}\log_2(|M|+1)$ to emphasize the highly oscillatory nature and scale of the function.

S22BB also accepts the parameters a and b as integral and decimal fractional components to increase the accuracy in the floating point calculations. This can provide a significant improvement to the solution when small

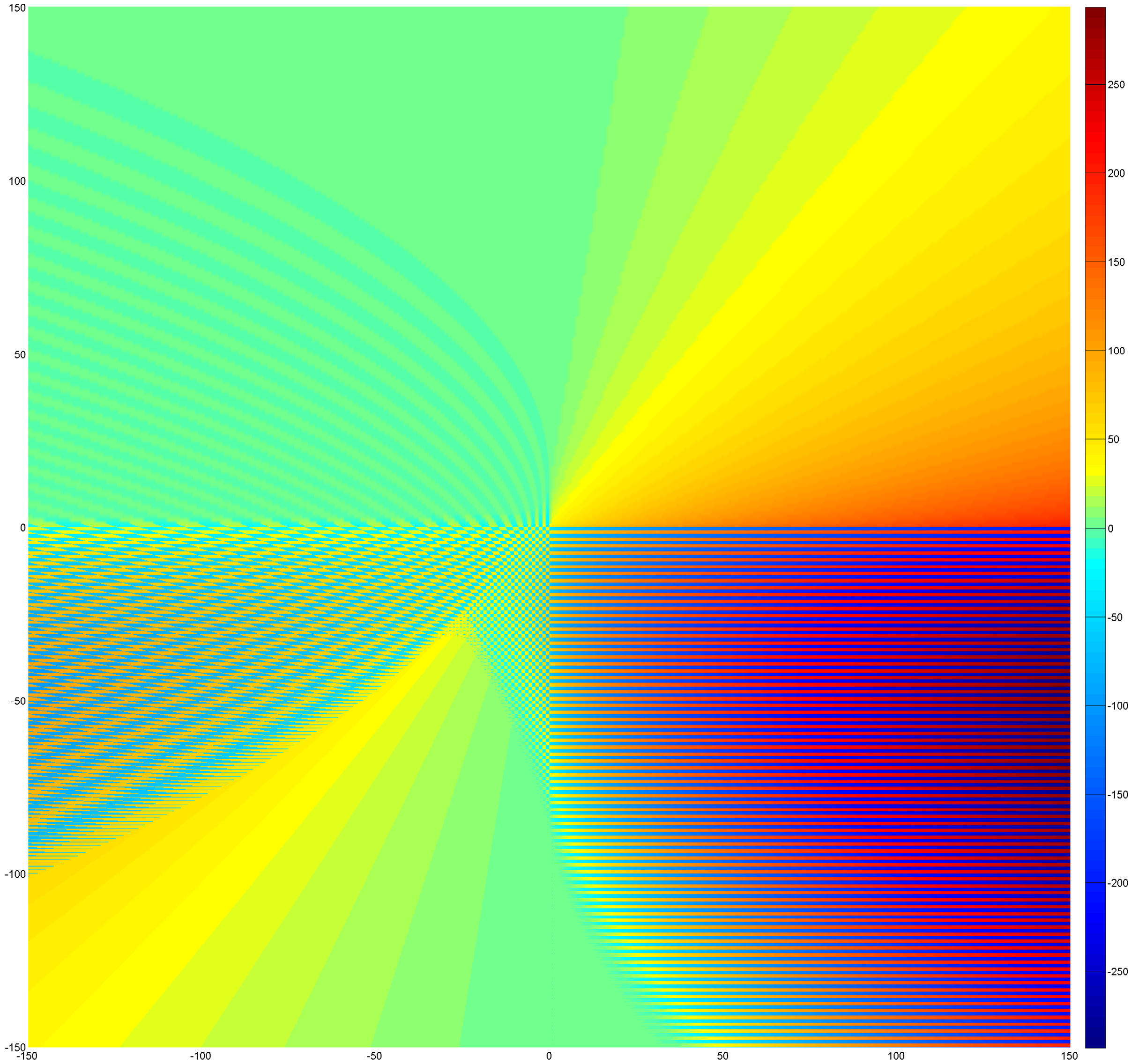

Figure 1: $M(a,b,x)$ for $a \in [-150, 150], b \in (-150, 150]$ and $x = 25$.

perturbations to integral values are required. For example, consider the solutions for $M(-199.999999, -400.00001, 600)$. S22BA gives $M(a,b,x) = -0.1320802726327450 \times 10^{295}$, whereas S22BB gives $M(a_{ni}+a_{dr}, b_{ni}+b_{dr}, x) = -0.1320803101722191 \times 10^{295}$, where the nearest integer and decimal remainder components are $a_{ni} = -200, a_{dr} = 10^{-6}, b_{ni} = -200$ and $b_{dr} = -10^{-6}$. The relative differences is $O(10^{-7})$.