NAG Library Routine Document

C09BAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

C09BAF computes the real, continuous wavelet transform in one dimension.

2 Specification

```
SUBROUTINE C09BAF (WAVNAM, WPARAM, N, X, NSCAL, SCALES, C, IFAIL)

INTEGER             WPARAM, N, NSCAL, SCALES(NSCAL), IFAIL
REAL (KIND=nag_wp) X(N), C(NSCAL,N)
CHARACTER(*)        WAVNAM
```

3 Description

C09BAF computes the real part of the one-dimensional, continuous wavelet transform

$$C_{s,k} = \int_{\mathbb{R}} x(t) \frac{1}{\sqrt{s}} \psi^* \left(\frac{t-k}{s} \right) dt,$$

of a signal $x(t)$ at scale s and position k, where the signal is sampled discretely at n equidistant points x_i, for $i = 1, 2, \ldots, n$. ψ is the wavelet function, which can be chosen to be the Morlet wavelet, the derivatives of a Gaussian or the Mexican hat wavelet ($*$ denotes the complex conjugate). The integrals of the scaled, shifted wavelet function are approximated and the convolution is then computed.

The mother wavelets supplied for use with this routine are defined as follows.

1. The Morlet wavelet (real part) with nondimensional wave number κ is

$$\psi(x) = \frac{1}{\pi^{1/4}} \left(\cos(\kappa x) - e^{-\kappa^2/2} \right) e^{-x^2/2},$$

where the correction term, $e^{-\kappa^2/2}$ (required to satisfy the admissibility condition) is included.

2. The derivatives of a Gaussian are obtained from

$$\hat{\psi}^{(m)}(x) = \frac{\mathrm{d}^m \left(e^{-x^2} \right)}{\mathrm{d}x^m},$$

taking $m = 1, \ldots, 8$. These are the Hermite polynomials multiplied by the Gaussian. The sign is then adjusted to give $\hat{\psi}^{(m)}(0) > 0$ when m is even while the sign of the succeeding odd derivative, $\hat{\psi}^{(m+1)}$, is made consistent with the preceding even numbered derivative. They are normalized by the L^2-norm,

$$p_m = \left(\int_{-\infty}^{\infty} \left[\hat{\psi}^{(m)}(x) \right]^2 \mathrm{d}x \right)^{1/2}$$

The resulting normalized derivatives can be written in terms of the Hermite polynomials, $H_m(x)$, as

$$\psi^{(m)}(x) = \frac{\alpha H_m(x) e^{-x^2}}{p_m},$$

where

$$\alpha = \begin{cases} 1, & \text{when } m = 0, 3 \bmod 4; \\ -1, & \text{when } m = 1, 2 \bmod 4. \end{cases}$$

Thus, the derivatives of a Gaussian provided here are,

$$\psi^{(1)}(x) = -\left(\frac{2}{\pi}\right)^{1/4} 2xe^{-x^2},$$

$$\psi^{(2)}(x) = -\left(\frac{2}{\pi}\right)^{1/4} \frac{1}{\sqrt{3}}\left(4x^2 - 2\right)e^{-x^2},$$

$$\psi^{(3)}(x) = \left(\frac{2}{\pi}\right)^{1/4} \frac{1}{\sqrt{15}}\left(8x^3 - 12x\right)e^{-x^2},$$

$$\psi^{(4)}(x) = \left(\frac{2}{\pi}\right)^{1/4} \frac{1}{\sqrt{105}}\left(16x^4 - 48x^2 + 12\right)e^{-x^2},$$

$$\psi^{(5)}(x) = -\left(\frac{2}{\pi}\right)^{1/4} \frac{1}{3\sqrt{105}}\left(32x^5 - 160x^3 + 120x\right)e^{-x^2},$$

$$\psi^{(6)}(x) = -\left(\frac{2}{\pi}\right)^{1/4} \frac{1}{3\sqrt{1155}}\left(64x^6 - 480x^4 + 720x^2 - 120\right)e^{-x^2},$$

$$\psi^{(7)}(x) = \left(\frac{2}{\pi}\right)^{1/4} \frac{1}{3\sqrt{15015}}\left(128x^7 - 1344x^5 + 3360x^3 - 1680x\right)e^{-x^2},$$

$$\psi^{(8)}(x) = \left(\frac{2}{\pi}\right)^{1/4} \frac{1}{45\sqrt{1001}}\left(256x^8 - 3584x^6 + 13440x^4 - 13440x^2 + 1680\right)e^{-x^2}.$$

3. The second derivative of a Gaussian is known as the Mexican hat wavelet and is supplied as an additional function in the form

$$\psi(x) = \frac{2}{\left(\sqrt{3}\pi^{1/4}\right)}\left(1 - x^2\right)e^{-x^2/2}.$$

The remaining normalized derivatives of a Gaussian can be expressed as multiples of the exponential $e^{-t^2/2}$ by applying the substitution $x = t/\sqrt{2}$ followed by multiplication with the scaling factor, $1/\sqrt[4]{2}$.

4 References

Daubechies I (1992) *Ten Lectures on Wavelets* SIAM, Philadelphia

5 Parameters

1: WAVNAM – CHARACTER(*) *Input*

On entry: the name of the mother wavelet. See the C09 Chapter Introduction for details.

WAVNAM = 'MORLET'
Morlet wavelet.

WAVNAM = 'DGAUSS'
Derivative of a Gaussian wavelet.

WAVNAM = 'MEXHAT'
Mexican hat wavelet.

Constraint: WAVNAM = 'MORLET', 'DGAUSS' or 'MEXHAT'.

2: WPARAM – INTEGER *Input*

On entry: the nondimensional wave number for the Morlet wavelet or the order of the derivative for the Gaussian wavelet. It is not referenced when WAVNAM = 'MEXHAT'.

Constraints:

if WAVNAM = 'MORLET', $5 \leq$ WPARAM ≤ 20;
if WAVNAM = 'DGAUSS', $1 \leq$ WPARAM ≤ 8.

3: N – INTEGER *Input*

On entry: the size, n, of the input dataset x.

Constraint: N ≥ 2.

4: X(N) – REAL (KIND=nag_wp) array *Input*

On entry: X contains the input dataset X$(j) = x_j$, for $j = 1, 2, \ldots, n$.

5: NSCAL – INTEGER *Input*

On entry: the dimension of the array SCALES and the first dimension of the array C as declared in the (sub)program from which C09BAF is called. The number of scales to be computed.

Constraint: NSCAL ≥ 1.

6: SCALES(NSCAL) – INTEGER array *Input*

On entry: the scales at which the transform is to be computed.

Constraint: SCALES$(i) \geq 1$, for $i = 1, 2, \ldots,$ NSCAL.

7: C(NSCAL,N) – REAL (KIND=nag_wp) array *Output*

On exit: the transform coefficients at the requested scales, where C(i, j) is the transform coefficient $C_{i,j}$ at scale i and position j.

8: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, WAVNAM $\neq$ 'MORLET', 'DGAUSS' or 'MEXHAT'

IFAIL = 2

On entry, WAVNAM = 'MORLET', and WPARAM < 5 or WPARAM > 20.
or WAVNAM = 'DGAUSS', and WPARAM < 1 or WPARAM > 8.

IFAIL = 3

On entry, N < 2.

IFAIL = 5

On entry, NSCAL < 1.

IFAIL = −999

Internal memory allocation failed.

7 Accuracy

The accuracy of C09BAF is determined by the fact that the convolution must be computed as a discrete approximation to the continuous form. The input signal, x, is taken to be piecewise constant using the supplied discrete values.

8 Further Comments

Workspace is internally allocated by C09BAF. The total size of these arrays is $2^{13} + (\mathrm{N} + n_k - 1)$ real elements and n_k integer elements, where $n_k = k \times \max(\mathrm{SCALES}(i))$ and $k = 17$ when WAVNAM = 'MORLET' or 'DGAUSS' and $k = 11$ when WAVNAM = 'MEXHAT'.

9 Example

This example computes the continuous wavelet transform of a dataset containing a single nonzero value representing an impulse. The Morlet wavelet is used with wave number $\kappa = 5$ and scales 1, 2, 3, 4.

9.1 Program Text

```
    Program c09bafe

!     C09BAF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: c09baf, nag_wp
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter                :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                           :: ifail, j, n, nscal, wparam
      Character (10)                    :: wavnam
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable   :: c(:,:), x(:)
      Integer, Allocatable              :: scales(:)
!     .. Intrinsic Procedures ..
      Intrinsic                         :: trim
!     .. Executable Statements ..
      Write (nout,*) 'C09BAF Example Program Results'
      Write (nout,*)
!     Skip heading in data file
      Read (nin,*)
!     Read problem parameters
      Read (nin,*) n, nscal
      Allocate (c(nscal,n),scales(nscal),x(n))
      Read (nin,*) wavnam, wparam
```

```
      Write (nout,99999) trim(wavnam), wparam, n, nscal

!     Read data array and write it out

      Read (nin,*) scales(1:nscal)
      Read (nin,*) x(1:n)

      Write (nout,99998) scales(1:nscal)
      Write (nout,99997) x(1:n)

!     ifail: behaviour on error exit
!             =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
      ifail = 0
      Call c09baf(wavnam,wparam,n,x,nscal,scales,c,ifail)

      Write (nout,99996) nscal
      Write (nout,99995) scales(1:nscal)
      Do j = 1, n
        Write (nout,99994) c(1:nscal,j)
      End Do

99999 Format (2X,'Parameters read from file ::'/4X,'Wavelet : ',A, &
        ',  wparam : ',I6/10X,'n : ',I6,',   nscal : ',I6)
99998 Format (/2X,'Input Data ::'/4X,' Scales :',5(I8,1X):)
99997 Format (5X,'     x :',5(F8.3,1X),(/13X,5(F8.3,1X)):)
99996 Format (/2X,'Number of Scales :      ',I10)
99995 Format (2X,'Wavelet coefficients C ::'/4X,'Scale :',I7,3I13)
99994 Format (10X,4(1P,E11.4,2X):)
    End Program c09bafe
```

9.2 Program Data

```
C09BAF Example Program Data
 10      4               : n, nscal
 MORLET  5               : wavnam
 1    2    3    4        : scales(1:nscal)
 0.0  0.0  0.0  0.0  1.0
 0.0  0.0  0.0  0.0  0.0 : x(1:n)
```

9.3 Program Results

```
 C09BAF Example Program Results

  Parameters read from file ::
    Wavelet : MORLET,  wparam :      5
          n :     10,   nscal :      4

  Input Data ::
     Scales :       1        2        3        4
          x :   0.000    0.000    0.000    0.000    1.000
                0.000    0.000    0.000    0.000    0.000

  Number of Scales :               4
  Wavelet coefficients C ::
    Scale :      1            2            3            4
          -1.7651E-05   1.5012E-04   5.2331E-02   1.4454E-01
          -1.3643E-03  -5.8141E-02   1.7057E-01  -8.4364E-02
           4.6511E-03   1.8442E-01  -1.4891E-01  -2.8870E-01
           8.9294E-02  -2.6380E-01  -2.6822E-01  -9.4993E-02
          -9.2563E-02   1.3289E-01   2.5680E-01   2.8293E-01
          -9.2563E-02   1.3289E-01   2.5680E-01   2.8293E-01
           8.9294E-02  -2.6380E-01  -2.6822E-01  -9.4993E-02
           4.6511E-03   1.8442E-01  -1.4891E-01  -2.8870E-01
          -1.3643E-03  -5.8141E-02   1.7057E-01  -8.4364E-02
          -1.7651E-05   1.5012E-04   5.2331E-02   1.4454E-01
```