

# NAG Library Routine Document

## F07GWF (ZPPTRI)

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

### 1 Purpose

F07GWF (ZPPTRI) computes the inverse of a complex Hermitian positive definite matrix  $A$ , where  $A$  has been factorized by F07GRF (ZPPTRF), using packed storage.

### 2 Specification

```
SUBROUTINE F07GWF (UPLO, N, AP, INFO)
```

```
INTEGER                N, INFO
COMPLEX (KIND=nag_wp) AP(*)
CHARACTER(1)          UPLO
```

The routine may be called by its LAPACK name *zpptri*.

### 3 Description

F07GWF (ZPPTRI) is used to compute the inverse of a complex Hermitian positive definite matrix  $A$ , the routine must be preceded by a call to F07GRF (ZPPTRF), which computes the Cholesky factorization of  $A$ , using packed storage.

If  $UPLO = 'U'$ ,  $A = U^H U$  and  $A^{-1}$  is computed by first inverting  $U$  and then forming  $(U^{-1})U^{-H}$ .

If  $UPLO = 'L'$ ,  $A = LL^H$  and  $A^{-1}$  is computed by first inverting  $L$  and then forming  $L^{-H}(L^{-1})$ .

### 4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

### 5 Parameters

- 1: UPLO – CHARACTER(1) *Input*  
*On entry:* specifies how  $A$  has been factorized.  
 UPLO = 'U'  
 $A = U^H U$ , where  $U$  is upper triangular.  
 UPLO = 'L'  
 $A = LL^H$ , where  $L$  is lower triangular.  
*Constraint:* UPLO = 'U' or 'L'.
- 2: N – INTEGER *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
*Constraint:*  $N \geq 0$ .

3: AP(\*) – COMPLEX (KIND=nag\_wp) array Input/Output

**Note:** the dimension of the array AP must be at least  $\max(1, N \times (N + 1)/2)$ .

*On entry:* the Cholesky factor of  $A$  stored in packed form, as returned by F07GRF (ZPPTRF).

*On exit:* the factorization is overwritten by the  $n$  by  $n$  matrix  $A^{-1}$ .

More precisely,

if UPLO = 'U', the upper triangle of  $A^{-1}$  must be stored with element  $A_{ij}$  in  $AP(i + j(j - 1)/2)$  for  $i \leq j$ ;

if UPLO = 'L', the lower triangle of  $A^{-1}$  must be stored with element  $A_{ij}$  in  $AP(i + (2n - j)(j - 1)/2)$  for  $i \geq j$ .

4: INFO – INTEGER Output

*On exit:* INFO = 0 unless the routine detects an error (see Section 6).

## 6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO =  $-i$ , the  $i$ th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO =  $i$ , the  $i$ th diagonal element of the Cholesky factor is zero; the Cholesky factor is singular and the inverse of  $A$  cannot be computed.

## 7 Accuracy

The computed inverse  $X$  satisfies

$$\|XA - I\|_2 \leq c(n)\epsilon\kappa_2(A) \quad \text{and} \quad \|AX - I\|_2 \leq c(n)\epsilon\kappa_2(A),$$

where  $c(n)$  is a modest function of  $n$ ,  $\epsilon$  is the *machine precision* and  $\kappa_2(A)$  is the condition number of  $A$  defined by

$$\kappa_2(A) = \|A\|_2 \|A^{-1}\|_2.$$

## 8 Further Comments

The total number of real floating point operations is approximately  $\frac{8}{3}n^3$ .

The real analogue of this routine is F07GJF (DPPTRI).

## 9 Example

This example computes the inverse of the matrix  $A$ , where

$$A = \begin{pmatrix} 3.23 + 0.00i & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 + 0.00i & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 + 0.00i & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 + 0.00i \end{pmatrix}.$$

Here  $A$  is Hermitian positive definite, stored in packed form, and must first be factorized by F07GRF (ZPPTRF).

## 9.1 Program Text

Program f07gwfe

```

!      F07GWF Example Program Text
!
!      Mark 24 Release. NAG Copyright 2012.
!
!      .. Use Statements ..
!      Use nag_library, Only: nag_wp, x04ddf, zpptrf, zpatri
!      .. Implicit None Statement ..
!      Implicit None
!      .. Parameters ..
!      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
!      Integer                    :: i, ifail, info, j, n
!      Character (1)              :: uplo
!      .. Local Arrays ..
!      Complex (Kind=nag_wp), Allocatable :: ap(:)
!      Character (1)              :: clabs(1), rlabs(1)
!      .. Executable Statements ..
!      Write (nout,*) 'F07GWF Example Program Results'
!      Skip heading in data file
!      Read (nin,*)
!      Read (nin,*) n

!      Allocate (ap(n*(n+1)/2))

!      Read A from data file

!      Read (nin,*) uplo
!      If (uplo=='U') Then
!         Read (nin,*)((ap(i+j*(j-1)/2),j=i,n),i=1,n)
!      Else If (uplo=='L') Then
!         Read (nin,*)((ap(i+(2*n-j)*(j-1)/2),j=1,i),i=1,n)
!      End If

!      Factorize A
!      The NAG name equivalent of zpptrf is f07grf
!      Call zpptrf(uplo,n,ap,info)

!      Write (nout,*)
!      Flush (nout)
!      If (info==0) Then

!         Compute inverse of A
!         The NAG name equivalent of zpatri is f07gwf
!         Call zpatri(uplo,n,ap,info)

!         Print inverse

!         ifail: behaviour on error exit
!                 =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
!         ifail = 0
!         Call x04ddf(uplo,'Nonunit',n,ap,'Bracketed','F7.4','Inverse', &
!                   'Integer',rlabs,'Integer',clabs,80,0,ifail)

!      Else
!         Write (nout,*) 'A is not positive definite'
!      End If

!      End Program f07gwfe

```

## 9.2 Program Data

F07GWF Example Program Data

```

4                                     :Value of N
'L'                                   :Value of UPLO
(3.23, 0.00)
(1.51, 1.92) ( 3.58, 0.00)
(1.90,-0.84) (-0.23,-1.11) ( 4.09, 0.00)
(0.42,-2.50) (-1.18,-1.37) ( 2.33, 0.14) ( 4.29, 0.00) :End of matrix A

```

## 9.3 Program Results

F07GWF Example Program Results

```

Inverse
          1          2          3          4
1 ( 5.4691, 0.0000)
2 (-1.2624,-1.5491) ( 1.1024, 0.0000)
3 (-2.9746,-0.9616) ( 0.8989,-0.5672) ( 2.1589, 0.0000)
4 ( 1.1962, 2.9772) (-0.9826,-0.2566) (-1.3756,-1.4550) ( 2.2934, 0.0000)

```

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