

NAG Library Routine Document

F07PWF (ZHPTRI)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F07PWF (ZHPTRI) computes the inverse of a complex Hermitian indefinite matrix A , where A has been factorized by F07PRF (ZHPTRF), using packed storage.

2 Specification

SUBROUTINE F07PWF (UPLO, N, AP, IPIV, WORK, INFO)

INTEGER N, IPIV(*), INFO
 COMPLEX (KIND=nag_wp) AP(*), WORK(N)
 CHARACTER(1) UPLO

The routine may be called by its LAPACK name *zhptri*.

3 Description

F07PWF (ZHPTRI) is used to compute the inverse of a complex Hermitian indefinite matrix A , the routine must be preceded by a call to F07PRF (ZHPTRF), which computes the Bunch–Kaufman factorization of A , using packed storage.

If UPLO = 'U', $A = PUDU^H P^T$ and A^{-1} is computed by solving $U^H P^T X P U = D^{-1}$ for X .

If UPLO = 'L', $A = PLDL^H P^T$ and A^{-1} is computed by solving $L^H P^T X P L = D^{-1}$ for X .

4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

- 1: UPLO – CHARACTER(1) *Input*
On entry: specifies how A has been factorized.
 UPLO = 'U'
 $A = PUDU^H P^T$, where U is upper triangular.
 UPLO = 'L'
 $A = PLDL^H P^T$, where L is lower triangular.
Constraint: UPLO = 'U' or 'L'.
- 2: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.

3: AP(*) – COMPLEX (KIND=nag_wp) array Input/Output

Note: the dimension of the array AP must be at least $\max(1, N \times (N + 1)/2)$.

On entry: the factorization of A stored in packed form, as returned by F07PRF (ZHPTRF).

On exit: the factorization is overwritten by the n by n matrix A^{-1} .

More precisely,

if UPLO = 'U', the upper triangle of A^{-1} must be stored with element A_{ij} in $AP(i + j(j - 1)/2)$ for $i \leq j$;

if UPLO = 'L', the lower triangle of A^{-1} must be stored with element A_{ij} in $AP(i + (2n - j)(j - 1)/2)$ for $i \geq j$.

4: IPIV(*) – INTEGER array Input

Note: the dimension of the array IPIV must be at least $\max(1, N)$.

On entry: details of the interchanges and the block structure of D , as returned by F07PRF (ZHPTRF).

5: WORK(N) – COMPLEX (KIND=nag_wp) array Workspace

6: INFO – INTEGER Output

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO = i , $d(i, i)$ is exactly zero; D is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies a bound of the form

if UPLO = 'U', $|DU^T P^T X P U - I| \leq c(n)\epsilon(|D||U^T|P^T|X|P|U| + |D||D^{-1}|)$;

if UPLO = 'L', $|DL^T P^T X P L - I| \leq c(n)\epsilon(|D||L^T|P^T|X|P|L| + |D||D^{-1}|)$,

$c(n)$ is a modest linear function of n , and ϵ is the *machine precision*.

8 Further Comments

The total number of real floating point operations is approximately $\frac{8}{3}n^3$.

The real analogue of this routine is F07PJF (DSPTRI).

9 Example

This example computes the inverse of the matrix A , where

$$A = \begin{pmatrix} -1.36 + 0.00i & 1.58 + 0.90i & 2.21 - 0.21i & 3.91 + 1.50i \\ 1.58 - 0.90i & -8.87 + 0.00i & -1.84 - 0.03i & -1.78 + 1.18i \\ 2.21 + 0.21i & -1.84 + 0.03i & -4.63 + 0.00i & 0.11 + 0.11i \\ 3.91 - 1.50i & -1.78 - 1.18i & 0.11 - 0.11i & -1.84 + 0.00i \end{pmatrix}.$$

Here A is Hermitian indefinite, stored in packed form, and must first be factorized by F07PRF (ZHPTRF).

9.1 Program Text

Program f07pwfe

```
!      F07PWF Example Program Text
!
!      Mark 24 Release. NAG Copyright 2012.
!
!      .. Use Statements ..
!      Use nag_library, Only: nag_wp, x04ddf, zhptraf, zhptri
!      .. Implicit None Statement ..
!      Implicit None
!      .. Parameters ..
!      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
!      Integer                    :: i, ifail, info, j, n
!      Character (1)              :: uplo
!      .. Local Arrays ..
!      Complex (Kind=nag_wp), Allocatable :: ap(:), work(:)
!      Integer, Allocatable        :: ipiv(:)
!      Character (1)              :: clabs(1), rlabs(1)
!      .. Executable Statements ..
!      Write (nout,*) 'F07PWF Example Program Results'
!      Skip heading in data file
!      Read (nin,*)
!      Read (nin,*) n

!      Allocate (ap(n*(n+1)/2),work(n),ipiv(n))

!      Read A from data file

!      Read (nin,*) uplo
!      If (uplo=='U') Then
!         Read (nin,*)((ap(i+j*(j-1)/2),j=i,n),i=1,n)
!      Else If (uplo=='L') Then
!         Read (nin,*)((ap(i+(2*n-j)*(j-1)/2),j=1,i),i=1,n)
!      End If

!      Factorize A
!      The NAG name equivalent of zhptraf is f07prf
!      Call zhptraf(uplo,n,ap,ipiv,info)

!      Write (nout,*)
!      Flush (nout)
!      If (info==0) Then

!         Compute inverse of A
!         The NAG name equivalent of zhptri is f07pwf
!         Call zhptri(uplo,n,ap,ipiv,work,info)

!         Print inverse

!         ifail: behaviour on error exit
!         =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
!         ifail = 0
!         Call x04ddf(uplo,'Nonunit',n,ap,'Bracketed','F7.4','Inverse', &
!           'Integer',rlabs,'Integer',clabs,80,0,ifail)
```

```

      Else
        Write (nout,*) 'The factor D is singular'
      End If

      End Program f07pwfe

```

9.2 Program Data

F07PWF Example Program Data

```

4                                     :Value of N
'L'                                   :Value of UPLO
(-1.36, 0.00)
( 1.58,-0.90) (-8.87, 0.00)
( 2.21, 0.21) (-1.84, 0.03) (-4.63, 0.00)
( 3.91,-1.50) (-1.78,-1.18) ( 0.11,-0.11) (-1.84, 0.00) :End of matrix A

```

9.3 Program Results

F07PWF Example Program Results

```

Inverse
      1           2           3           4
1 ( 0.0826, 0.0000)
2 (-0.0335, 0.0440) (-0.1408, 0.0000)
3 ( 0.0603,-0.0105) ( 0.0422,-0.0222) (-0.2007, 0.0000)
4 ( 0.2391,-0.0926) ( 0.0304, 0.0203) ( 0.0982,-0.0635) ( 0.0073,-0.0000)

```
