

Unscented Kalman Filter

State space models have applications in a wide range of fields including economics [4] and control engineering, and are frequently used in the machine learning [2] and time series [1] literature.

One common method for applying a state space model is the Kalman filter. Routines for analysing linear state space models via the Kalman filter were introduced into the NAG Library at Mark 17 (see, for example, G13EA and G13EB). This functionality has been extended to cover nonlinear state space models with the introduction of G13EJ and G13EK at Mark 25.

There are two commonly used algorithms for applying a Kalman filter to a nonlinear state space model: the Extended Kalman Filter (EKF) and the Unscented Kalman Filter (UKF). Both of the new routines mentioned above apply the UKF, as described in [3], to a nonlinear state space model with additive noise which, at time t , can be described by:

$$\begin{aligned}x_{t+1} &= F(x_t) + v_t \\ y_t &= H(x_t) + u_t\end{aligned}$$

where x_t represents the unobserved state vector of length m_x and y_t the observed measurement vector of length m_y . The process noise is denoted v_t , which is assumed to have mean zero and covariance structure Σ_x , and the measurement noise by u_t , which is assumed to have mean zero and covariance structure Σ_y . F and H are known, possibly nonlinear, functions.

As a simple illustration of a nonlinear state space model, consider the movement of a hypothetical robot. We define

$$\begin{aligned}x_{t+1} &= (\xi_{t+1} \quad \eta_{t+1} \quad \theta_{t+1})^T \\ &= F(x_t) + v_t \\ &= x_t + \begin{pmatrix} \cos \theta_t & -\sin \theta_t & 0 \\ \sin \theta_t & \cos \theta_t & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.5r & 0.5r \\ 0 & 0 \\ r/d & -r/d \end{pmatrix} \begin{pmatrix} \phi_{Rt} \\ \phi_{Lt} \end{pmatrix} + v_t\end{aligned}$$

where x_{t+1} is the position and orientation of the robot (with respect to a reference frame) at time $t + 1$, with (ξ, η) giving the x and y coordinates and θ the angle (with respect to the x -axis) that the robot is facing. The robot has two drive wheels, of radius r on an axle of length d . During time period t the right wheel is believed to rotate at a velocity of ϕ_{Rt} and the left at a velocity of ϕ_{Lt} . In this example, these velocities are fixed with $\phi_{Rt} = 0.4$ and $\phi_{Lt} = 0.1$. The state update function, F , calculates where the robot should be at each time point, given its previous position. However, in reality, there is some random fluctuation in the velocity of the wheels, for example, due to slippage. Therefore the actual position of the robot and the position given by equation F will differ. The magnitude of these random fluctuations is included in the model through Σ_x .

In the area that the robot is moving there is a single wall. The position of the wall is known and defined by its distance, Δ , from the origin and its angle, A , from the x -axis. The robot has a sensor that is able to measure, δ , the distance to the wall and α the angle to the wall. A measurement function H can then be constructed to give the expected distance and angle to the wall if the robot's position is given by x_t . The measurement vector y and measurement

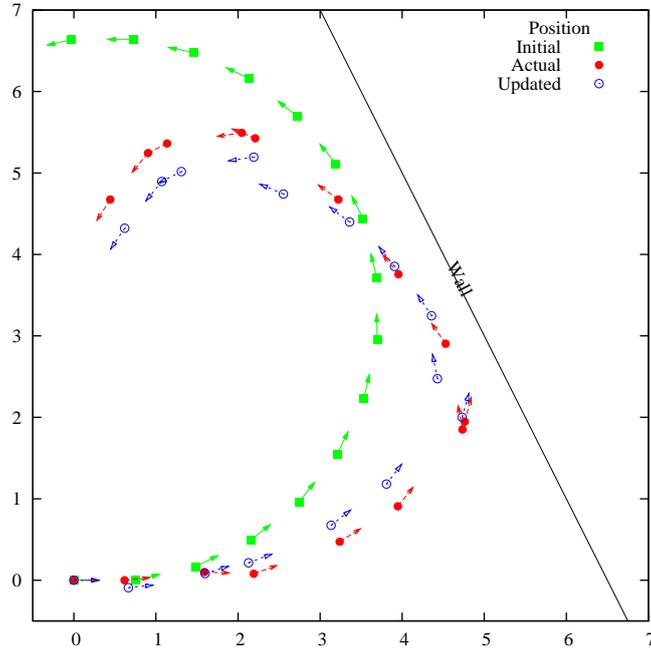


Figure 1: An illustration of the position and orientation of a hypothetical robot. The theoretical position of the robot, if no wheel slippage occurred is denoted by ‘initial’. The actual position of the robot is denoted by ‘actual’ and ‘updated’ shows an estimate of the robots position calculated by combining information on the theoretical wheel rotation and readings from an error prone sensor.

function H are given by:

$$\begin{aligned}
 y_t &= (\delta_t, \alpha_t)^T \\
 &= H(x_t) + u_t \\
 &= \begin{pmatrix} \Delta - \xi_t \cos A - \eta_t \sin A \\ \theta_t - A \end{pmatrix} + u_t
 \end{aligned}$$

The accuracy of the sensor is subject to random fluctuations, denoted u_t above, and the magnitude of these fluctuations is included in the model through Σ_y .

This state space model therefore allows the robot’s position at time t , x_t to be estimated by combining information on the rotation of the wheels (encoded in F) and the sensor information y .

Rather than directly supplying the covariance matrices Σ_x and Σ_y , we use the lower Cholesky factorizations, L_x and L_y where

$$L_x = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix}, \quad L_y = \begin{pmatrix} 0.01 & 0 \\ 0 & 0.01 \end{pmatrix}$$

where $L_x L_x^T = \Sigma_x$ and $L_y L_y^T = \Sigma_y$. Initial values for the state vector and covariance matrix were taken to be:

$$x_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, P_0 = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix}$$

The results from applying the unscented Kalman filter to this model can be seen in Figure [1](#).

References

- [1] A C Harvey. *Forecasting, Structural Time Series Models and the Kalman Filter*. Cambridge University Press, 1989.
- [2] S Haykin. *Kalman Filtering and Neural Networks*. John Wiley and Sons, 2001.
- [3] S J Julier and J K Uhlmann. A new extension of the Kalman Filter to nonlinear systems. *International Symposium for Aerospace/Defense, Sensing, Simulation and Controls (Volume 3)*, 26, 1997.
- [4] V Martin, S Hurn, and D Harris. *Econometric Modelling with Time Series: Specification, Estimation and Testing*. Cambridge University Press, 2013.