

## NAG Library Function Document

### nag\_deviates\_students\_t (g01fbc)

#### 1 Purpose

nag\_deviates\_students\_t (g01fbc) returns the deviate associated with the given tail probability of Student's  $t$ -distribution with real degrees of freedom.

#### 2 Specification

```
#include <nag.h>
#include <nagg01.h>

double nag_deviates_students_t (Nag_TailProbability tail, double p, double df,
                               NagError *fail)
```

#### 3 Description

The deviate,  $t_p$  associated with the lower tail probability,  $p$ , of the Student's  $t$ -distribution with  $\nu$  degrees of freedom is defined as the solution to

$$P(T < t_p : \nu) = p = \frac{\Gamma((\nu + 1)/2)}{\sqrt{\nu\pi}\Gamma(\nu/2)} \int_{-\infty}^{t_p} \left(1 + \frac{T^2}{\nu}\right)^{-(\nu+1)/2} dT, \quad \nu \geq 1; -\infty < t_p < \infty.$$

For  $\nu = 1$  or  $2$  the integral equation is easily solved for  $t_p$ .

For other values of  $\nu < 3$  a transformation to the beta distribution is used and the result obtained from nag\_deviates\_beta (g01fec).

For  $\nu \geq 3$  an inverse asymptotic expansion of Cornish–Fisher type is used. The algorithm is described by Hill (1970).

#### 4 References

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

Hill G W (1970) Student's  $t$ -distribution *Comm. ACM* **13**(10) 617–619

#### 5 Arguments

1: **tail** – Nag\_TailProbability *Input*

*On entry:* indicates which tail the supplied probability represents.

**tail** = Nag\_UpperTail

The upper tail probability, i.e.,  $P(T \geq t_p : \nu)$ .

**tail** = Nag\_LowerTail

The lower tail probability, i.e.,  $P(T \leq t_p : \nu)$ .

**tail** = Nag\_TwoTailSignif

The two tail (significance level) probability, i.e.,  $P(T \geq |t_p| : \nu) + P(T \leq -|t_p| : \nu)$ .

**tail** = Nag\_TwoTailConfid

The two tail (confidence interval) probability, i.e.,  $P(T \leq |t_p| : \nu) - P(T \leq -|t_p| : \nu)$ .

*Constraint:* **tail** = Nag\_UpperTail, Nag\_LowerTail, Nag\_TwoTailSignif or Nag\_TwoTailConfid.

- 2: **p** – double *Input*  
*On entry:*  $p$ , the probability from the required Student's  $t$ -distribution as defined by **tail**.  
*Constraint:*  $0.0 < \mathbf{p} < 1.0$ .
- 3: **df** – double *Input*  
*On entry:*  $\nu$ , the degrees of freedom of the Student's  $t$ -distribution.  
*Constraint:*  $\mathbf{df} \geq 1.0$ .
- 4: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 3.6 in the Essential Introduction).

## 6 Error Indicators and Warnings

On any of the error conditions listed below except NE\_SOL\_NOT\_CONV nag\_deviates\_students\_t (g01fbc) returns 0.0.

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

### NE\_REAL\_ARG\_GE

On entry,  $\mathbf{p} = \langle value \rangle$ .  
Constraint:  $\mathbf{p} < 1.0$ .

### NE\_REAL\_ARG\_LE

On entry,  $\mathbf{p} = \langle value \rangle$ .  
Constraint:  $\mathbf{p} > 0.0$ .

### NE\_REAL\_ARG\_LT

On entry,  $\mathbf{df} = \langle value \rangle$ .  
Constraint:  $\mathbf{df} \geq 1.0$ .

### NE\_SOL\_NOT\_CONV

The solution has failed to converge. However, the result should be a reasonable approximation.

## 7 Accuracy

The results should be accurate to five significant digits, for most argument values. The error behaviour for various argument values is discussed in Hill (1970).

## 8 Further Comments

The value  $t_p$  may be calculated by using the transformation described in Section 3 and using nag\_deviates\_beta (g01fec). This function allows you to set the required accuracy.

## 9 Example

This example reads the probability, the tail that probability represents and the degrees of freedom for a number of Student's  $t$ -distributions and computes the corresponding deviates.

### 9.1 Program Text

```

/* nag_deviates_students_t (g01fbc) Example Program.
 *
 * Copyright 1996 Numerical Algorithms Group.
 *
 * Mark 4, 1996.
 * Mark 5 revised, 1998.
 * Mark 7 revised, 2001.
 *
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
    Integer          exit_status=0;
    double           df, p, t;
    int              i;
    static Nag_TailProbability tail[] = { Nag_LowerTail, Nag_UpperTail,
                                         Nag_TwoTailSignif, Nag_TwoTailConfid };
    static const char *tailmess[] = { "Nag_LowerTail", "Nag_UpperTail",
                                       "Nag_TwoTailSignif",
                                       "Nag_TwoTailConfid" };

    NagError         fail;

    INIT_FAIL(fail);

    printf(
        "nag_deviates_students_t (g01fbc) Example Program Results\n\n");
    /* Skip heading in data file */
    scanf("%*[\n]");
    printf("      p      df      tail      t\n\n");
    while (scanf("%lf %lf %d", &p, &df, &i) != EOF)
    {
        /* nag_deviates_students_t (g01fbc).
         * Deviates for Student's t-distribution
         */
        t = nag_deviates_students_t(tail[i], p, df, &fail);
        if (fail.code != NE_NOERROR)
        {
            printf("Error from nag_deviates_students_t (g01fbc).\n%s\n",
                fail.message);
            exit_status = 1;
            goto END;
        }
        printf("%8.3f%8.3f  %-19s  %8.3f\n", p, df, tailmess[i], t);
    }

    END:
    return exit_status;
}

```

### 9.2 Program Data

```

nag_deviates_students_t (g01fbc) Example Program Data
0.0100  20.0  2
0.01    7.5  0
0.99   45.0  3

```

### 9.3 Program Results

nag\_deviates\_students\_t (g01fbc) Example Program Results

p	df	tail	t
0.010	20.000	Nag_TwoTailSignif	2.845
0.010	7.500	Nag_LowerTail	-2.943
0.990	45.000	Nag_TwoTailConfid	2.690

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