

NAG Library Function Document

nag_cwt_1d_real (c09bac)

1 Purpose

nag_cwt_1d_real (c09bac) computes the real, continuous wavelet transform in one dimension.

2 Specification

```
#include <nag.h>
#include <nagc09.h>

void nag_cwt_1d_real (Nag_Wavelet wavnam, Integer wparam, Integer n,
    const double x[], Integer nscal, const Integer scales[], double c[],
    NagError *fail)
```

3 Description

nag_cwt_1d_real (c09bac) computes the real part of the one-dimensional, continuous wavelet transform

$$C_{s,k} = \int_{\mathbb{R}} x(t) \frac{1}{\sqrt{s}} \psi^* \left(\frac{t-k}{s} \right) dt,$$

of a signal $x(t)$ at scale s and position k , where the signal is sampled discretely at n equidistant points x_i , for $i = 1, 2, \dots, n$. ψ is the wavelet function, which can be chosen to be the Morlet wavelet, the derivatives of a Gaussian or the Mexican hat wavelet (* denotes the complex conjugate). The integrals of the scaled, shifted wavelet function are approximated and the convolution is then computed.

The mother wavelets supplied for use with this function are defined as follows.

1. The Morlet wavelet (real part) with nondimensional wave number κ is

$$\psi(x) = \frac{1}{\pi^{1/4}} \left(\cos(\kappa x) - e^{-\kappa^2/2} \right) e^{-x^2/2},$$

where the correction term, $e^{-\kappa^2/2}$ (required to satisfy the admissibility condition) is included.

2. The derivatives of a Gaussian are obtained from

$$\hat{\psi}^{(m)}(x) = \frac{d^m \left(e^{-x^2} \right)}{dx^m},$$

taking $m = 1, \dots, 8$. These are the Hermite polynomials multiplied by the Gaussian. The sign is then adjusted to give $\hat{\psi}^{(m)}(0) > 0$ when m is even while the sign of the succeeding odd derivative, $\hat{\psi}^{(m+1)}$, is made consistent with the preceding even numbered derivative. They are normalized by the L^2 -norm,

$$p_m = \left(\int_{-\infty}^{\infty} \left[\hat{\psi}^{(m)}(x) \right]^2 dx \right)^{1/2}$$

The resulting normalized derivatives can be written in terms of the Hermite polynomials, $H_m(x)$, as

$$\psi^{(m)}(x) = \frac{\alpha H_m(x) e^{-x^2}}{p_m},$$

where

$$\alpha = \begin{cases} 1, & \text{when } m = 0, 3 \bmod 4; \\ -1, & \text{when } m = 1, 2 \bmod 4. \end{cases}$$

Thus, the derivatives of a Gaussian provided here are,

$$\psi^{(1)}(x) = -\left(\frac{2}{\pi}\right)^{1/4} 2xe^{-x^2},$$

$$\psi^{(2)}(x) = -\left(\frac{2}{\pi}\right)^{1/4} \frac{1}{\sqrt{3}}(4x^2 - 2)e^{-x^2},$$

$$\psi^{(3)}(x) = \left(\frac{2}{\pi}\right)^{1/4} \frac{1}{\sqrt{15}}(8x^3 - 12x)e^{-x^2},$$

$$\psi^{(4)}(x) = \left(\frac{2}{\pi}\right)^{1/4} \frac{1}{\sqrt{105}}(16x^4 - 48x^2 + 12)e^{-x^2},$$

$$\psi^{(5)}(x) = -\left(\frac{2}{\pi}\right)^{1/4} \frac{1}{3\sqrt{105}}(32x^5 - 160x^3 + 120x)e^{-x^2},$$

$$\psi^{(6)}(x) = -\left(\frac{2}{\pi}\right)^{1/4} \frac{1}{3\sqrt{1155}}(64x^6 - 480x^4 + 720x^2 - 120)e^{-x^2},$$

$$\psi^{(7)}(x) = \left(\frac{2}{\pi}\right)^{1/4} \frac{1}{3\sqrt{15015}}(128x^7 - 1344x^5 + 3360x^3 - 1680x)e^{-x^2},$$

$$\psi^{(8)}(x) = \left(\frac{2}{\pi}\right)^{1/4} \frac{1}{45\sqrt{1001}}(256x^8 - 3584x^6 + 13440x^4 - 13440x^2 + 1680)e^{-x^2}.$$

- The second derivative of a Gaussian is known as the Mexican hat wavelet and is supplied as an additional function in the form

$$\psi(x) = \frac{2}{(\sqrt{3}\pi^{1/4})}(1 - x^2)e^{-x^2/2}.$$

The remaining normalized derivatives of a Gaussian can be expressed as multiples of the exponential $e^{-t^2/2}$ by applying the substitution $x = t/\sqrt{2}$ followed by multiplication with the scaling factor, $1/\sqrt[4]{2}$.

4 References

Daubechies I (1992) *Ten Lectures on Wavelets* SIAM, Philadelphia

5 Arguments

1: **wavnam** – Nag_Wavelet

Input

On entry: the name of the mother wavelet. See the c09 Chapter Introduction for details.

wavnam = Nag_Morlet
Morlet wavelet.

wavnam = Nag_DGauss
Derivative of a Gaussian wavelet.

wavnam = Nag_MexHat
Mexican hat wavelet.

Constraint: **wavnam** = Nag_Morlet, Nag_DGauss or Nag_MexHat.

- 2: **wparam** – Integer *Input*
On entry: the nondimensional wave number for the Morlet wavelet or the order of the derivative for the Gaussian wavelet. It is not referenced when **wavnam** = Nag_MexHat.
Constraints:
 if **wavnam** = Nag_Morlet, $5 \leq \mathbf{wparam} \leq 20$;
 if **wavnam** = Nag_DGauss, $1 \leq \mathbf{wparam} \leq 8$.
- 3: **n** – Integer *Input*
On entry: the size, n , of the input dataset x .
Constraint: $\mathbf{n} \geq 2$.
- 4: **x[n]** – const double *Input*
On entry: **x** contains the input dataset $\mathbf{x}[j - 1] = x_j$, for $j = 1, 2, \dots, n$.
- 5: **nscal** – Integer *Input*
On entry: the number of scales to be computed.
Constraint: $\mathbf{nscal} \geq 1$.
- 6: **scales[nscal]** – const Integer *Input*
On entry: the scales at which the transform is to be computed.
Constraint: $\mathbf{scales}[i - 1] \geq 1$, for $i = 1, 2, \dots, \mathbf{nscal}$.
- 7: **c[nscal × n]** – double *Output*
Note: the (i, j) th element of the matrix C is stored in $\mathbf{c}[(j - 1) \times \mathbf{nscal} + i - 1]$.
On exit: the transform coefficients at the requested scales, where $\mathbf{c}[(j - 1) \times \mathbf{nscal} + i - 1]$ is the transform coefficient $C_{i,j}$ at scale i and position j .
- 8: **fail** – NagError * *Input/Output*
 The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_BAD_PARAM

On entry, argument $\langle \text{value} \rangle$ had an illegal value.

NE_INT

On entry, $\mathbf{n} = \langle \text{value} \rangle$.
 Constraint: $\mathbf{n} \geq 2$.

On entry, **nscal** = $\langle value \rangle$.

Constraint: **nscal** ≥ 1 .

On entry, **wavnam** = Nag_DGauss and **wparam** = $\langle value \rangle$.

Constraint: if **wavnam** = Nag_DGauss, $1 \leq \mathbf{wparam} \leq 8$.

On entry, **wavnam** = Nag_Morlet and **wparam** = $\langle value \rangle$.

Constraint: if **wavnam** = Nag_Morlet, $5 \leq \mathbf{wparam} \leq 20$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

7 Accuracy

The accuracy of nag_cwt_1d_real (c09bac) is determined by the fact that the convolution must be computed as a discrete approximation to the continuous form. The input signal, x , is taken to be piecewise constant using the supplied discrete values.

8 Parallelism and Performance

Not applicable.

9 Further Comments

Workspace is internally allocated by nag_cwt_1d_real (c09bac). The total size of these arrays is $2^{13} + (\mathbf{n} + n_k - 1)$ double elements and n_k Integer elements, where $n_k = k \times \max(\mathbf{scales}[i - 1])$ and $k = 17$ when **wavnam** = Nag_Morlet or Nag_DGauss and $k = 11$ when **wavnam** = Nag_MexHat.

10 Example

This example computes the continuous wavelet transform of a dataset containing a single nonzero value representing an impulse. The Morlet wavelet is used with wave number $\kappa = 5$ and scales 1, 2, 3, 4.

10.1 Program Text

```

/* nag_cwt_1d_real (c09bac) Example Program.
 *
 * Copyright 2011, Numerical Algorithms Group.
 *
 * Mark 23, 2011.
 */
#include <nag.h>
#include <nag_stdlib.h>
#include <nagc09.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    Integer    exit_status = 0;
    Integer    i, n, nscal, wparam;
    /* Arrays */
    Integer    *scales = 0;
    double     *c = 0, *x = 0;
    char       *clabs = 0, **clabsc = 0;
    char       wavnam[20];
    /* NAG types */
    NagError   fail;
    Nag_Wavelet wavnamenum;

    INIT_FAIL(fail);

```

```

printf("nag_cwt_ld_real (c09bac) Example Program Results\n\n");

/* Skip heading in data file*/
scanf("%*[\n]");
/* Read problem parameters*/
scanf("%ld%ld", &n, &nscal);
scanf("%*[\n]");
if (
    !(c = NAG_ALLOC((nscal)*(n), double)) ||
    !(scales = NAG_ALLOC((nscal), Integer)) ||
    !(x = NAG_ALLOC((n), double)) ||
    !(clabs = NAG_ALLOC(10*10, char)) ||
    !(clabsc = NAG_ALLOC(10, char *))
)
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}
scanf("%19s", wavnam);
/* nag_enum_name_to_value (x04nac).
 * Converts NAG enum member name to value
 */
wavnamenum = (Nag_Wavelet) nag_enum_name_to_value(wavnam);
scanf("%ld%*[\n]", &wparam);
printf("Parameters read from file ::\n");
printf("    Wavelet : %20s, wparam : %ld\n", wavnam, wparam);
printf("        n : %20ld, nscal : %ld\n", n, nscal);

/* Read data array and write it out*/
for (i = 0; i < nscal; i++) scanf("%" NAG_IFMT "", &scales[i]);
scanf("%*[\n] ");
for (i = 0; i < n; i++) scanf("%lf", &x[i]);
scanf("%*[\n] ");
printf("Input Data ::\n");
printf("    Scales :");
for (i = 0; i < nscal; i++) printf("%9ld", scales[i]);

printf("\n        x :");
for (i = 0; i < n; i++)
    printf("%9.3f%s", x[i], (i+1)%5 ? "" : "\n        ");
printf("\n");
/* nag_cwt_ld_real (c09bac).
 * One-dimensional real continuous wavelet transform
 */
nag_cwt_ld_real(wavnamenum, wparam, n, x, nscal, scales, c, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_cwt_ld_real (c09bac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

printf("Number of Scales :%9ld\n\n", nscal);
fflush(stdout);

/* print coefficients for each scale as columns of a matrix. This requires
 * printing c as a row major matrix with principal dimension nscal using
 * nag_gen_real_mat_print_comp (x04cbc).
 */
for (i = 0; i < nscal; i++)
    sprintf(&clabs[10*i], "scale %3ld", scales[i]);
for (i = 0; i < nscal; i++) clabsc[i] = &clabs[i*10];
nag_gen_real_mat_print_comp(Nag_RowMajor, Nag_GeneralMatrix, Nag_NonUnitDiag,
    n, nscal, c, nscal, "%13.4e",
    "Wavelet coefficients C ::", Nag_NoLabels, 0,
    Nag_CharacterLabels, (const char **) clabsc, 80,
    0,
    NULL,
    &fail);
if (fail.code != NE_NOERROR)

```

```

    {
      printf("Error from nag_gen_real_mat_print_comp (x04cbc).\n%s\n",
            fail.message);
      exit_status = 6;
      goto END;
    }

  END:
  NAG_FREE(c);
  NAG_FREE(x);
  NAG_FREE(scales);
  NAG_FREE(clabs);
  NAG_FREE(clabsc);
  return exit_status;
}

```

10.2 Program Data

```

nag_cwt_ld_real (c09bac) Example Program Data
10      4      : n, nscal
Nag_Morlet      5      : wavnam, wparam
1      2      3      4      : scales(1:nscal)
0.0  0.0  0.0  0.0  1.0
0.0  0.0  0.0  0.0  0.0 : x[]

```

10.3 Program Results

```

nag_cwt_ld_real (c09bac) Example Program Results

```

```

Parameters read from file ::

```

```

  Wavelet :      Nag_Morlet, wparam : 5
    n :      10, nscal : 4

```

```

Input Data ::

```

```

  Scales :      1      2      3      4
    x :      0.000    0.000    0.000    0.000    1.000
           0.000    0.000    0.000    0.000    0.000

```

```

Number of Scales :      4

```

```

Wavelet coefficients C ::

```

```

  scale  1      scale  2      scale  3      scale  4
-1.7651e-05  1.5012e-04  5.2331e-02  1.4454e-01
-1.3643e-03  -5.8141e-02  1.7057e-01  -8.4364e-02
 4.6511e-03  1.8442e-01  -1.4891e-01  -2.8870e-01
 8.9294e-02  -2.6380e-01  -2.6822e-01  -9.4993e-02
-9.2563e-02  1.3289e-01  2.5680e-01  2.8293e-01
-9.2563e-02  1.3289e-01  2.5680e-01  2.8293e-01
 8.9294e-02  -2.6380e-01  -2.6822e-01  -9.4993e-02
 4.6511e-03  1.8442e-01  -1.4891e-01  -2.8870e-01
-1.3643e-03  -5.8141e-02  1.7057e-01  -8.4364e-02
-1.7651e-05  1.5012e-04  5.2331e-02  1.4454e-01

```
