

NAG Library Function Document

nag_bessel_i_alpha (s18ejc)

1 Purpose

nag_bessel_i_alpha (s18ejc) returns a sequence of values for the modified Bessel functions $I_{\alpha+n-1}(x)$ or $I_{\alpha-n+1}(x)$ for real x , non-negative $\alpha < 1$ and $n = 1, 2, \dots, |N| + 1$.

2 Specification

```
#include <nag.h>
#include <nags.h>

void nag_bessel_i_alpha (double x, double a, Integer nl, Complex b[],
                        NagError *fail)
```

3 Description

nag_bessel_i_alpha (s18ejc) evaluates a sequence of values for the modified Bessel function of the first kind $I_\alpha(x)$, where x is real and nonzero and α is the order with $0 \leq \alpha < 1$. The $(|N| + 1)$ -member sequence is generated for orders $\alpha, \alpha + 1, \dots, \alpha + N$ when $N \geq 0$. Note that $+$ is replaced by $-$ when $N < 0$. For positive orders the function may also be called with $x = 0$, since $I_q(0) = 0$ when $q > 0$. For negative orders the formula

$$I_{-q}(x) = I_q(x) + \frac{2}{\pi} \sin(\pi q) K_q(x)$$

is used to generate the required sequence.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

- | | | |
|----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------|
| 1: | x – double
<i>On entry:</i> the argument x of the function.
<i>Constraint:</i> if nl < 0, x ≠ 0.0. | Input |
| 2: | a – double
<i>On entry:</i> the order α of the first member in the required sequence of function values.
<i>Constraint:</i> $0.0 \leq \mathbf{a} < 1.0$. | Input |
| 3: | nl – Integer
<i>On entry:</i> the value of N .
<i>Constraint:</i> $\text{abs}(\mathbf{nl}) \leq 101$. | Input |
| 4: | b [\times] – Complex
<i>On exit:</i> with fail.code = NE_NOERROR or fail.code = NW_SOME_PRECISION_LOSS, the required sequence of function values: b (n) contains $I_{\alpha+n-1}(x)$ if nl ≥ 1 and $I_{\alpha-n+1}(x)$ otherwise, for $n = 1, 2, \dots, \text{abs}(\mathbf{nl}) + 1$. | Output |

5: **fail** – NagError *

Input/Output

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **nl** = $\langle value \rangle$.
Constraint: $\text{abs}(\mathbf{nl}) \leq 101$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_OVERFLOW_LIKELY

The evaluation has been abandoned due to the likelihood of overflow.

NE_REAL

On entry, **a** = $\langle value \rangle$.
Constraint: $0.0 \leq \mathbf{a} < 1.0$.

NE_REAL_INT

On entry, **x** = $\langle value \rangle$, **nl** = $\langle value \rangle$.
Constraint: $\mathbf{x} \neq 0.0$ when $\mathbf{nl} < 0$.

NE_TERMINATION_FAILURE

The evaluation has been abandoned due to failure to satisfy the termination condition.

NE_TOTAL_PRECISION_LOSS

The evaluation has been abandoned due to total loss of precision.

NW_SOME_PRECISION_LOSS

The evaluation has been completed but some precision has been lost.

7 Accuracy

All constants in the underlying functions are specified to approximately 18 digits of precision. If t denotes the number of digits of precision in the floating point arithmetic being used, then clearly the maximum number of correct digits in the results obtained is limited by $p = \min(t, 18)$. Because of errors in argument reduction when computing elementary functions inside the underlying functions, the actual number of correct digits is limited, in general, by $p - s$, where $s \approx \max(1, |\log_{10}|x||, |\log_{10}|\alpha||)$ represents the number of digits lost due to the argument reduction. Thus the larger the values of $|x|$ and $|\alpha|$, the less the precision in the result.

8 Further Comments

None.

9 Example

The example program evaluates $I_0(x)$, $I_1(x)$, $I_2(x)$ and $I_3(x)$ at $x = 0.5$, and prints the results.

9.1 Program Text

```

/* nag_bessel_i_alpha (s18ejc) Example Program.
 *
 * Copyright 2000 Numerical Algorithms Group.
 *
 * NAG C Library
 *
 * Mark 6, 2000.
 * Mark 8 revised, 2004.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Complex *b = 0;
    Integer exit_status = 0, i, nl;
    NagError fail;
    double a, alpha, d, x;

    INIT_FAIL(fail);

    /* Skip heading in data file */
    scanf("%*[\n]");
    printf("nag_bessel_i_alpha (s18ejc) Example Program Results\n");
    if (!(b = NAG_ALLOC(101, Complex)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
    while (scanf("%lf %lf %ld%*[\n]", &x, &a, &nl) != EOF)
    {
        printf(" x      a      nl\n");
        printf("%4.1f %4.1f %6ld\n\n", x, a, nl);
        /* nag_bessel_i_alpha (s18ejc).
         * Modified Bessel functions  $I_{\alpha+n-1}(x)$  or
         *  $I_{\alpha-n+1}(x)$  for real  $x \neq 0$ , non-negative
         *  $\alpha < 1$  and  $n = 1, 2, \dots, |N|+1$ 
         */
        nag_bessel_i_alpha(x, a, nl, b, &fail);
        if (fail.code == NE_NOERROR)
        {
            printf(" Requested values of  $I_{\alpha}(X)$ \n\n");
            alpha = a;
            printf("      alpha       $I_{\alpha}(X)$ \n");
            for (i = 1; i <= ABS(nl) + 1; ++i)
            {
                printf("%13.4e (%13.4e, %13.4e)\n",
                    alpha, b[i - 1].re, b[i - 1].im);
                d = (double) nl;
                alpha += SIGN(1.0, d);
            }
        }
        else
        {
            printf("Error from nag_bessel_i_alpha (s18ejc).\n%s\n",
                fail.message);
            exit_status = 1;
            goto END;
        }
    }
    END:
    if (b) NAG_FREE(b);
    return exit_status;
}
/* main */

```

9.2 Program Data

nag_bessel_i_alpha (s18ejc) Example Program Data
0.5 0.0 3 : Values of x, a and nl

9.3 Program Results

nag_bessel_i_alpha (s18ejc) Example Program Results

x a nl
0.5 0.0 3

Requested values of I_alpha(X)

alpha	I_alpha(X)	
0.0000e+00	(1.0635e+00,	0.0000e+00)
1.0000e+00	(2.5789e-01,	0.0000e+00)
2.0000e+00	(3.1906e-02,	0.0000e+00)
3.0000e+00	(2.6451e-03,	0.0000e+00)
